EFFECT OF EXTERNAL HEAT EXCHANGE ON THE IMPREGNATION OF A PREHEATED FILLER

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The kinetics of the penetration of a viscous liquid inside a heated porous body under conditions of nonideal heat transfer into the liquid is investigated.

The impregnation of a heated filamentary filler drawn through a chamber filled with a highly viscous liquid is an important process in the manufacture of certain composition materials. As a result of the contact with the filler, the liquid surrounding it is heated up and its viscosity decreases considerably, which facilitates the penetration of the liquid inside the filler due to the action of the pressure drop p" — p' (Fig. 1). A mathematical description of this process requires the solution of an extremely nonlinear problem on combined heat transfer and filtering. This problem has been investigated in [1] in the limiting case of ideally rapid heat and liquid transfer into the neighborhood of the transported filler (achieved, e.g., due to intense mixing of the liquid), when the temperature of the liquid can be regarded as uniform and equal to T" and only the processes inside the filler filled with liquid (region III in Fig. 1) need to be considered. It turned out, in particular, that in this case the length L_x which is impregnated does not depend on the thermal characteristics of the materials.

In reality no particular measures are usually taken to ensure that mixing of the liquid occurs, and natural mixing of the liquid due to convection is difficult because of its high viscosity. In this case the temperature on the boundary $y = 0$ of the filler differs from T" and the rate of the impregnation process may depend to a considerable extent on the thermal conductivity in the pure liquid (region II in Fig. 1). Obviously, any additional "resistance" to heat transfer from the filler to the surrounding liquid must lead to an increase in the mean temperature inside region III and to a corresponding reduction in the mean viscosity of the filtering liquid, and, consequently, to an acceleration of the impregnation process.

Strictly speaking, heat transfer into the filler, whose pores are filled with gas (region I in Fig. 1) may also affect the filtering of the liquid. However, its effect, although it may, of course, lead to some change in the form of the impregnation front Y(x), is not very important on the whole, since this process, which leads to some redistribution of the local heat fluxes, does not change the overall convective flow of heat in the region $x > 0$, transferred by the filler, heated to a high temperature T'. Hence, to a first approximation we can assume, as in [1], that the temperature at $y = Y(x)$ is equal to $T' > T''$.

For simplicity, we will consider the plane problem of the impregnation of a single strip of filler, using the same assumptions regarding the small relaxation time of the interphase heat and momentum transfer in region III, as in [1]. Then, using the results of [1], we will write the following approximate equation of convective heat transfer in region III:

$$u\frac{\partial T}{\partial x} + \varkappa v(x)\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2}, \quad 0 \leqslant y \leqslant Y(x), \tag{1}$$

where

$$a = \frac{\lambda_y}{\varepsilon\rho_0 c_0 + (1-\varepsilon)\rho_1 c_1}; \quad \varkappa = \frac{\rho_0 c_0}{\varepsilon\rho_0 c_0 + (1-\varepsilon)\rho_1 c_1}, \tag{2}$$

while v(x) is the component of the liquid filtering velocity in the y direction. In this case we have the following relations [1]:

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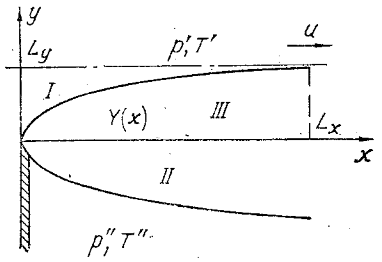

Fig. 1

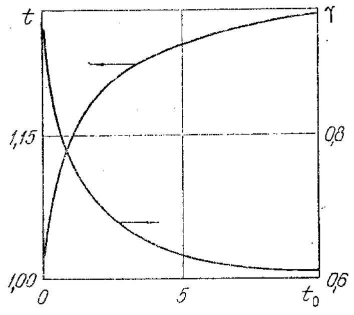

Fig. 2

Fig. 1. Sketch illustrating the process investigated: region I) filler with pores filled with gas; II) pure liquid in which the thermal boundary layer is formed; and III) filler with pores filled with liquid.

Fig. 2. Dependence of the parameter t and the coefficient of the reduction in the impregnation length γ on t_0 when $\delta = 1$.

$$v(x) = \varepsilon u \frac{dY(x)}{dx}, \quad p_0 = p'' - p' + p_c = \frac{v(x)}{\alpha} \int_0^{Y(x)} \mu(T)\, dy, \tag{3}$$

where T can be regarded as function of x and y.

The temperature in region II must also be found by solving the convective heat transfer equation. Since the Prandtl number is usually much greater than unity in liquids, for actual processes when the velocity of motion of the filler u is not small, we need only investigate this equation for large Peclét numbers. In this case, since the Prandtl number is high, we can assume that the thermal boundary layer is "submerged" in the dynamic boundary layer, i.e., we can approximate the components of the velocity of the liquid in the x and y directions by the quantities u and v(x), respectively, in the thermal boundary layer at the surface of the filler. Hence, in region II we have the following equation [2]:

$$u \frac{\partial \tau}{\partial x} + v(x) \frac{\partial \tau}{\partial y} = b \frac{\partial^2 \tau}{\partial y^2}, \quad y < 0, \tag{4}$$

where $b = \lambda/\rho_0 c_0$ is the thermal diffusivity of the liquid.

The boundary conditions for Eqs. (1) and (4) have the form

$$\begin{gathered} T = T', \quad y = Y(x); \quad \tau \to T'', \quad y \to -\infty, \\ T = \tau, \quad \lambda_y \frac{\partial T}{\partial y} = \lambda \frac{\partial \tau}{\partial y}, \quad y = 0. \end{gathered} \tag{5}$$

The unknown boundary Y(x) of the region in which T is defined and the coefficient of v(x) in (1) and (4) depend nonlinearly on the required temperature field and are formally defined by relations (3).

The results obtained in [1] suggest that in this case it is natural to seek a self-similar solution of the problem. The introduction of the self-similar variables h and η in accordance with the definitions

$$h = \left(\frac{u}{2a}\right)^{1/2} \frac{y}{\sqrt{x}}, \quad \eta = \left(\frac{u}{2b}\right)^{1/2} \frac{y}{\sqrt{x}}, \tag{6}$$

and also the functions

$$Y(x) = \left(\frac{u}{2a}\right)^{-1/2} H\sqrt{x}, \quad v(x) = \varepsilon \left(\frac{ua}{2}\right)^{1/2} \frac{H}{\sqrt{x}} \tag{7}$$

in a form which satisfies the first relation in (3), reduces the problem to the form

$$\frac{d^2T}{dh^2} = (\varkappa H - h) \frac{dT}{dh}, \quad 0 \leqslant h \leqslant H,$$

$$\frac{d^2\tau}{d\eta^2}=\left[\left(\varkappa\,\frac{\lambda_y}{\lambda}\right)^{1/2}H-\eta\right]\frac{d\tau}{d\eta},\quad \eta<0,$$
$$T=T',\quad h=H;\quad \tau\to T'',\quad \eta\to-\infty, \tag{8}$$
$$T=\tau,\quad \frac{dT}{dh}=\left(\varkappa\,\frac{\lambda}{\lambda_y}\right)^{1/2}\frac{d\tau}{d\eta},\quad h=\eta=0.$$

The parameter H in (7) and (8), considered here as a constant quantity, can be found, as in [1], from the equation

$$p_0=\frac{\varepsilon a H}{\alpha}\int\limits_0^H \mu(T)\,dh, \tag{9}$$

which follows from the second relation in (3) and the closing problem.

The solution of problem (8) can be represented in the form

$$T=C\int\limits_0^h \exp\left(-\frac{h^2}{2}+\varkappa Hh\right)dh+B, \tag{10}$$

$$\tau=C\left(\frac{1}{\varkappa}\,\frac{\lambda_y}{\lambda}\right)^{1/2}\int\limits_0^\eta \exp\left[-\frac{\eta^2}{2}+\left(\varkappa\,\frac{\lambda_y}{\lambda}\right)H\eta\right]d\eta+B,$$

where the constants of integration are

$$C=(T'-T'')\,[\mathrm{I}_1(\varkappa,\ H)+\mathrm{I}_2(\varkappa,\ \lambda_y/\lambda,\ H)]^{-1},$$
$$B=T'-C\mathrm{I}_1(\varkappa,\ H), \tag{11}$$
$$\mathrm{I}_1(\varkappa,\ H)=\int\limits_0^H \exp\left(-\frac{t^2}{2}+\varkappa Ht\right)dt,$$
$$\mathrm{I}_2(\varkappa,\ \lambda_y/\lambda,\ H)=\left(\frac{1}{\varkappa}\,\frac{\lambda_y}{\lambda}\right)^{1/2}\int\limits_0^\infty \exp\left[-\frac{t^2}{2}-\left(\varkappa\,\frac{\lambda_y}{\lambda}\right)^{1/2}Ht\right]dt.$$

All the integrals in (11) can be expressed in terms of known functions using tabulated integrals [3].

The case of slow impregnation is of particular interest. This occurs (see [1]) when

$$h\lesssim H\ll 1,\quad \mathrm{I}_1(\varkappa,\ H)\approx H. \tag{12}$$

We then have from (10) and (11)

$$T\approx T'-(T'-T'')\,\frac{H-h}{H_*}, \tag{13}$$

where we have introduced an effective value H_* of the parameter H:

$$H_*=H+\mathrm{I}_2(\varkappa,\ \lambda_y/\lambda,\ H)=H+H_0\exp(\beta^2)\,\mathrm{erfc}(\beta),$$
$$\beta=\left(\varkappa\,\frac{\lambda_y}{\lambda}\right)^{1/2}H,\quad H_0=\left(\frac{\pi}{2\varkappa}\,\frac{\lambda_y}{\lambda}\right)^{1/2},\quad \mathrm{erfc}(x)=\frac{2}{\sqrt{\pi}}\int\limits_x^\infty e^{t^2}\,dt. \tag{14}$$

If λ_y does not exceed some λ, then bearing (12) in mind, $\beta \ll 1$ and we can use the representation for the function in (14) when $\beta\to 0$, i.e.,

$$H_*\approx H+H_0. \tag{15}$$

The case of an ideally heat conducting liquid considered in [1] corresponds to passage to the limit $\lambda_y/\lambda\to 0$, $H_*\to H$. In general, obviously $H_*>H$, so that the temperature inside the filler exceeds the value calculated in [1], which facilitates, as already mentioned, the penetration of the liquid into the filler. In particular, on the surface of the filler

$$T_s\approx T''+(T'-T'')\,\frac{H_0}{H_*}>T''. \tag{16}$$

The second limiting case, the opposite of that investigated in [1], occurs when $\lambda_y/\lambda \gg H^2$. This corresponds to the situation when the thermal conductivity of the filler material is of the same order of magnitude or greater than the thermal conductivity of the liquid. In this important practical case we can assume that

$$H_* \approx H_0, \quad T = T' = \text{const}, \tag{17}$$

which corresponds to filtering of a liquid at constant temperature, i.e., when the viscosity is independent of the coordinates.

The solution obtained above for the temperature field inside the filler is similar to the solution obtained in [1] if we replace the temperature T'' of the liquid in the latter by the higher temperature T_s on the filler surface. Using further the different functional dependence of the viscosity of the liquid on the temperature we can evaluate the integral in (9) and determine H, after which it is easy to obtain the fundamental quantity characterizing the process from the practical point of view, viz., the length L_x subjected to impregnation.

We will consider the same versions of the approximation of the function $\mu(T)$ as in [1]. Suppose

$$\mu(T) \approx \mu_0 \left(1 - \frac{T - T''}{T_0}\right). \tag{18}$$

Here we obtain the equation for H in the form

$$\left(1 - \frac{T - T''}{T_0} \frac{H_0 + H/2}{H_*}\right) H^2 = \frac{\alpha p_0}{\varepsilon a \mu_0}. \tag{19}$$

The approximation

$$\mu(T) \approx \mu_0 \exp\left(-\frac{T - T''}{T_0}\right) \tag{20}$$

leads to the equation

$$HH_* \exp\left(-\frac{T' - T''}{T_0} \frac{H_0}{H_*}\right) \left[1 - \exp\left(-\frac{T' - T''}{T_0}\right)\right] = \frac{\alpha p_0}{\varepsilon a \mu_0} \frac{T' - T''}{T_0}. \tag{21}$$

Finally, the functional relation

$$\mu(T) \approx \mu_0 \exp \frac{T_0}{T} \tag{22}$$

corresponds to the following equation for H:

$$HH_* F(H) = \frac{\alpha p_0}{\varepsilon a \mu_0}, \quad E_i(x) = \int_{-\infty}^{x} \frac{e^t}{t}\, dt,$$
$$F(H) = \frac{T'}{T_0} \exp \frac{T_0}{T'} - \frac{T_s}{T_0} \exp \frac{T_0}{T_s} - E_i\left(\frac{T_0}{T_s}\right) + E_i\left(\frac{T_0}{T'}\right), \tag{23}$$

where T_s depends on H and is defined in (16). When $H \to 0$ the equations obtained above reduce to those given in [1].

Having the dependence of H on the process parameters, the impregnation length L_x can be found from the relation obtained from (7):

$$L_x = \frac{u}{2a} \left(\frac{L_y}{H}\right)^2. \tag{24}$$

It is convenient to describe the effect of the external heat exchange on the rate of impregnation using the coefficient γ of the reduction in the impregnation length, which is the ratio of the value of L_x, calculated from (24), to the similar quantity obtained in the limit as $H_0 \to 0$. (The latter quantity is found independently in [1].) For example, if we approximate the temperature dependence of the viscosity using (20), we have

$$\gamma = 1/t^2, \tag{25}$$

where t is the root of the equation

$$t(t+t_0)\exp\left(-\frac{\delta(t_0)}{t+t_0}\right)\left[1-\exp\left(-\frac{\delta t}{t+t_0}\right)\right]=1-e^{-\delta}, \tag{26}$$

which depends on the parameters

$$\delta=\frac{T''-T'}{T_0},\quad t_0=\left(\frac{\pi}{2\varkappa}\,\frac{\lambda_y}{\lambda}\,\frac{\varepsilon a\mu_0}{\delta\alpha p_0}\right)^{1/2}(1-e^{-\delta})^{1/2}. \tag{27}$$

The first of these parameters represents the degree of reduction of the viscosity as the temperature of the liquid increases from T" to T', and the second represents the relative effect of the slowing down of the heat-transfer process from the filler to the surrounding liquid on the acceleration of the impregnation.

Curves of t and γ as a function of t_0 for $\delta=1$ (which corresponds to the situation when $\mu(T')$ is e times less than $\mu(T'')$) are shown in Fig. 2. They convincingly confirm the fact that a deterioration in the heat exchange in the pure liquid can lead to a considerable reduction in the impregnated length, i.e. in practice, to an intensification of the corresponding process. In this case the values of L_x corresponding to an ideally heat conducting liquid can be regarded as the maximum corresponding to the most unsuitable conditions.

NOTATION

T, τ, temperature; p, pressure; u, pulling velocity; Y(x), thickness of the boundary layer; x, y, longitudinal and transverse coordinate; v, filtering velocity of the liquid; a, $\varkappa$, coefficients introduced in (2); p_0, effective pressure drop taking the capillary pressure p_c into account; α, penetrability of the filler; $\mu(t)$, viscosity of the liquid; μ_0, characteristic value of the viscosity occurring in the approximation relations for the viscosity; $b=\lambda/\rho_0 c_0$, thermal diffusivity of the liquid; ρ_0, ρ_1, density of the liquid and filler material; h, η, self-similar variables introduced in (6); c_0, specific heat capacity of the liquid; λ, λ_y, thermal conductivities; H, parameter defined from Eq. (9); H_*, effective value of the parameter H; ε, porosity of the filler; B, C, constants of integration defined in (11); I_1, I_2, functions defined in (11); β, H_0, quantities defined in (14); T_s, temperature of the filler surface; L_x, L_y, characteristic dimensions; T_0, a quantity occurring in the approximation relations for the viscosity; F, a function in (23); γ, a coefficient defined in (25); t, root of the equation in (26); δ, t_0, quantities defined in (27); ', parameters, relating to the region I; and ", parameters relating to region II.

LITERATURE CITED

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